Calculate the value of π

Rakesh Goel
CEO, SK Dynamics P. Ltd., B5 Industrial Estate, Roorkee 247 667

I. Abstract: π is a constant extensively used in Mathematics and application of mathematics in Physics etc. In fact, π is considered to be the most important constant in Physics. However, a simple way to calculate π is not available in standard math's textbooks of XII Class or lower whereas π is used right from VIII class onwards in the calculation of area of circles etc. The author has devised a simple way based on First, Second, Successive approximation method to calculate the value of π. The method simply uses the Pythagoras Theorem and the property of an equilateral triangle to calculate the value of π by definition which is the ratio of the circumference of the circle to its diameter. The paper also includes a software code which uses this algorithm to calculate the value of π.

II. Calculating the value of π:

By definition:

$$\pi = \frac{\text{circumference of a circle}}{\text{Diameter of the circle}}$$

$$= \frac{2\pi r}{2r}$$

$$= \pi$$

First Approximation:

Assume a circle of unit radius (radius = 1) and divide it into six segments. Now join segment lines at intersection of circle and diameter lines of 60 degrees each. This makes 6 equilateral triangles within the circle.

As a first approximation, the sum of 6 segments is considered as the circumference of the circle.

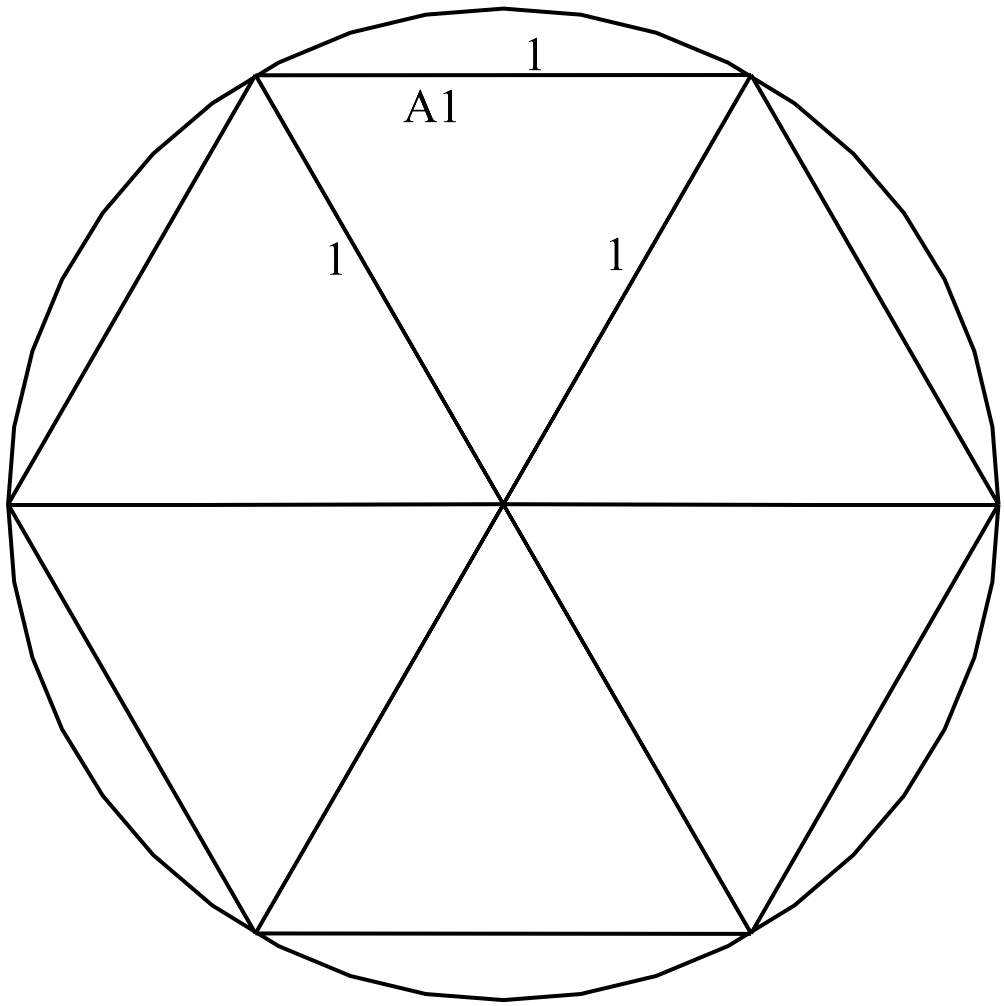
Referring to Figure 1:

π = Circumference/ Diameter

= (6*A1) /(2*1) { now A1 = 1 because it is a side of the equilateral triangle}

= 6/ 2

= 3



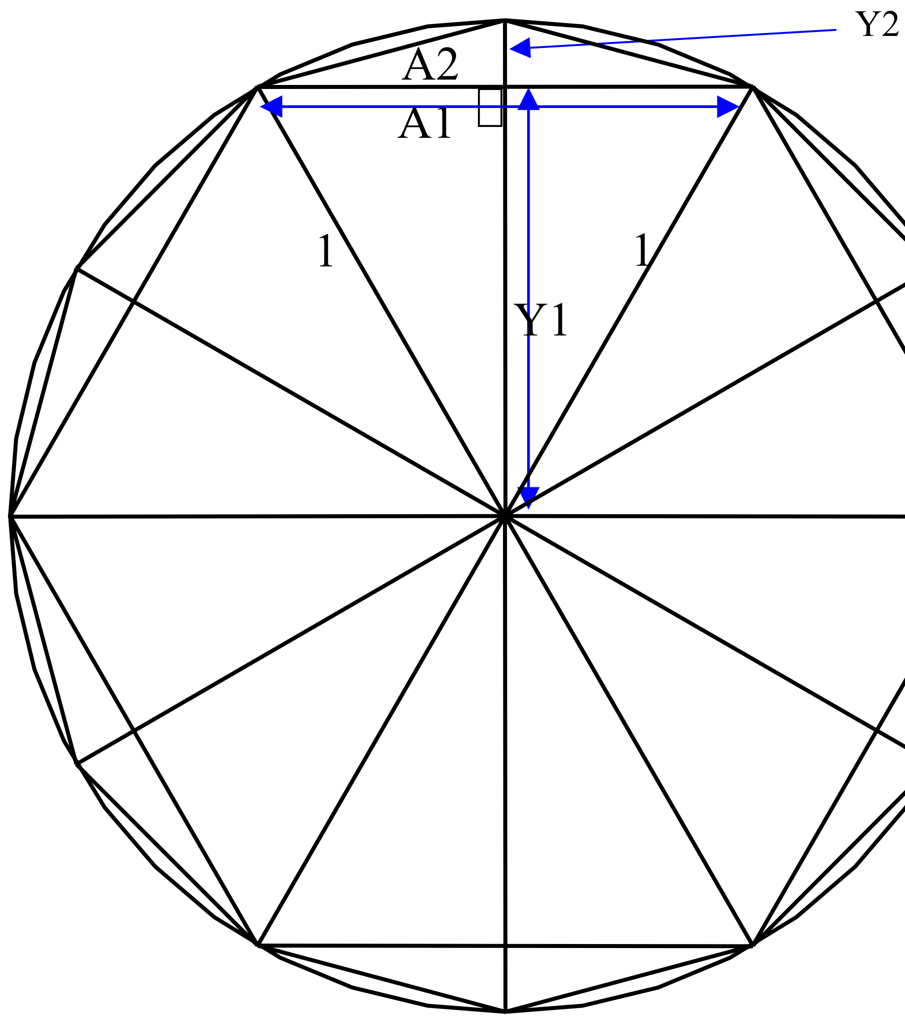
First Approximation
Fig 1

Second Approximation:

This circle is also having unit radius but divided into twelve segments.

We know that A1 = 1 from the Fig 1 (First approximation method).

Y1 + Y2 = 1 where Y2 , (A1)/2 and A2 form the sides of a right triangle and so also Y1, (A1)/2 and 1 form the sides of a larger right triangle as shown in Fig 2. Now the value of A2 is calculated as follows:



Second Approximation
Fig 2

$Y1^2 + (A1^2)/4 = 1$
$Y1^2 = 1 - \frac{1}{4}$

$Y1^2 = ¾$

$Y1 = 0.8660$

Therefore,

$Y2 = 1 – 0.8660$

$= 0.134$

Further,

$A2^2 = ((A1)/2)^2 + Y2^2$

$= 1/4 + (0.134)^2$

$= 0.267956$

$A2 = 0.51764466$

Now,

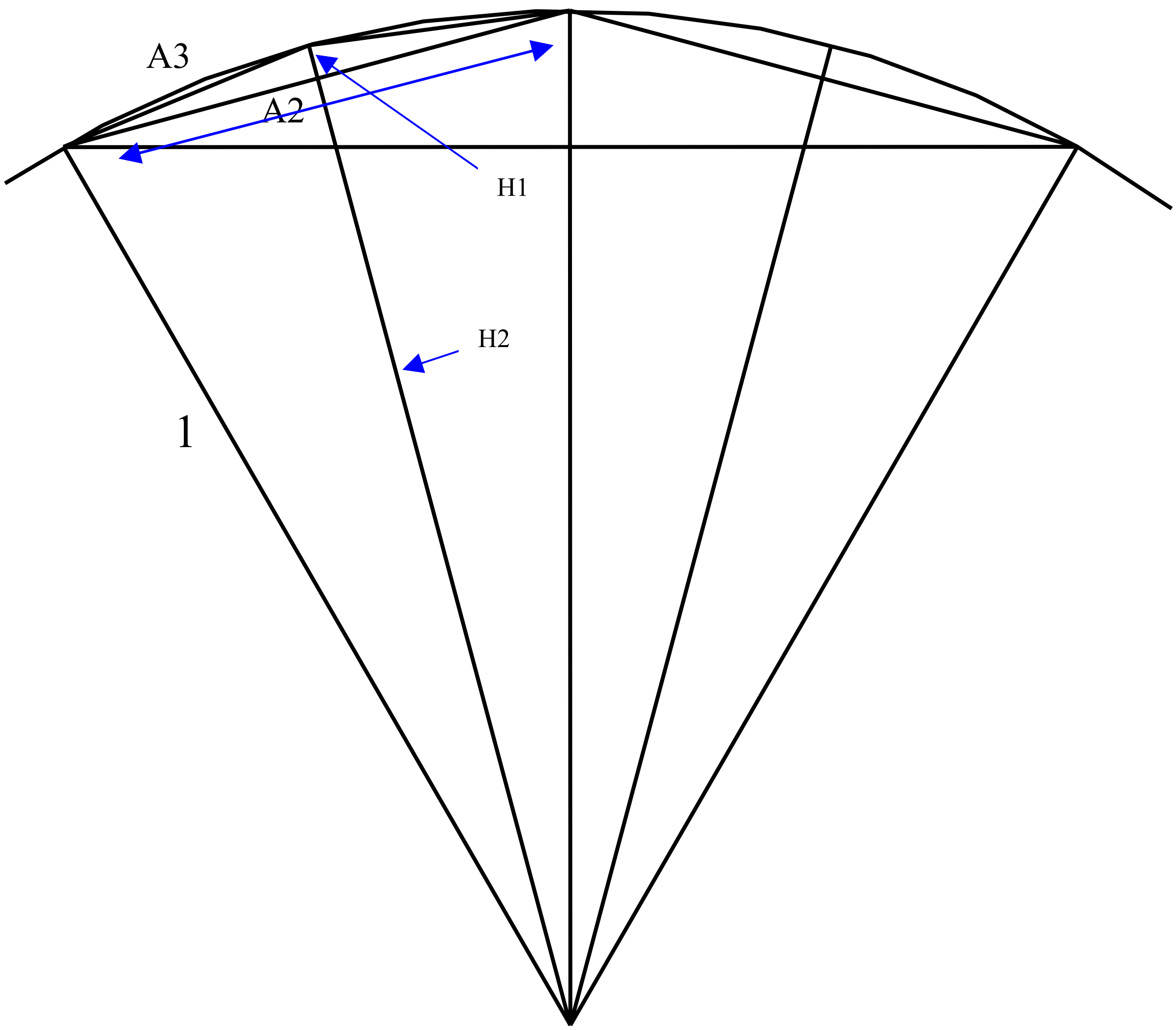
π = Circumference/ Diameter { The sum of equivalent segments is the approximation of the circumference where A2 is one of the 12 equivalent segments}

$= (12*A2) /(2*1)$

$= 3.10586796$

Third Approximation:

The unit circle is divided into 24 segments. One sixth portion of the circle is shown in Fig 3.



Third/ Successive Approximation
Fig 3

H1 + H2 = 1

$H2^2 \quad = \quad 1 - ((A2)/2)^2$

$\quad = \quad 1 - (0.51764466/ 2)^2$

= 0.933011001

H2 = 0.965924946

Therefore,

H1 = 1 - 0.965924946

= 0.034075053902

Now,

$A3^2$ = $((A2)/ 2)^2 + (0.034075053902)^2$

= $(0.51764466/ 2)^2 +$ 0.0011611092

= 0.066988998 + 0.0011611092

= 0.0681501077

A3 = 0.2610557559

Now,

π = Circumference/ Diameter

= (24*A3) /(2*1)

= 3.13266907121

III. Successive Approximation Method

This is a general method for better accuracy for the value of π and is an extension of the approximation methods seen above. (The circle can be further divided into 48, 96 etc segments to get more and more accuracy for the value of π). The same Fig 3 is referred here.

From the Third Approximation method:

$A3 = \sqrt{ \{ ((A2)/2)^2 + [1 - \sqrt{(1 - ((A2)/2)^2)}]^2 \} }$

Generalizing:

$A(J+1) = \sqrt{ \{ (A(J)/2)^2 + [1 - \sqrt{(1 - (A(J)/2)^2)}]^2 \} }$ ----------- Equation 1

Thus,

$\pi = 2^{(J-1)} * 3 * A(J)$ ----------------- Equation 2

where A(1) = 1

e.g.:

Consider J = 3

A(1) = 1

$A(2) = \sqrt{ \{ (A(1)/2)^2 + [1 - \sqrt{(1 - (A(1)/2)^2)}]^2 \} }$ from Equation 1

= 0.5176446658

$A(3) = \sqrt{ \{ (A(2)/2)^2 + [1 - \sqrt{(1 - (A(2)/2)^2)}]^2 \} }$ from Equation 1

= 0.2610557590

$\pi = 2^2 * 3 * A(3)$ from Equation 2

= 12 * 0.2610557590

= 3.132669108

which confirms the general relation obtained when we compare with value of π obtained from the Third Approximation Method.

IV. Source Code in QBasic to calculate the value of π using Successive approximation method

```
DEFDBL A-Z
        ' Calculating value of PI by Successive Approximation
        ' A(1)=1
        ' PI = (2 ^ (J - 1)) * 3 * A(J)
        ' A(J+1)=   SQR( (1-SQR(1-(A(J)/2)^2))^2 + (A(J)/2)^2 )
        CLS
        NN = 26   ' Number of loops
        A = 1
        FOR J = 1 TO NN
        PI = (2 ^ (J - 1)) * 3 * A
        PRINT J, A, PI
        X1 = (A / 2) * (A / 2)
        X2 = SQR(1 - X1)
        X3 = (1 - X2) * (1 - X2)
        A = SQR(X3 + X1)
        NEXT J
```

Results (values of π obtained by running the above code)

No. of loops (J)	A(J)	π PI
1	1	3
2	.5176380902050416	3.10582854123025
3	.2610523844401032	3.132628613281239
4	.1308062584602862	3.139350203046868
5	.0654381656435523	3.14103195089051
6	3.272346325297357D-02	3.141452472285463
7	1.636227920787426D-02	3.141557607911858
8	8.181208052469581D-03	3.141583892148319
9	4.090612582328191D-03	3.141590463228051
10	2.04530736067661D-03	3.141592105999273
11	1.022653814027395D-03	3.141592516692159
12	5.113269237248348D-04	3.141592619365385
13	2.556634639513095D-04	3.141592645033692
14	1.278317322367663D-04	3.141592651450769
15	6.391586615102209D-05	3.141592653055038
16	3.195793307959091D-05	3.141592653456105
17	1.597896654030544D-05	3.141592653556372
18	7.989483270216469D-06	3.141592653581439
19	3.994741635116202D-06	3.141592653587705
20	1.997370817559097D-06	3.141592653589272
21	9.986854087796732D-07	3.141592653589663
22	4.993427043898521D-07	3.141592653589762
23	2.49671352194928D-07	3.141592653589786
24	1.248356760974643D-07	3.141592653589792
25	6.241783804873215D-08	3.141592653589794
26	3.120891902436608D-08	3.141592653589794

The value of π remains unchanged after the 25th approximation,. The resolution of the computer software is limited to 19 digits. Further, accurate values of π can be obtained using any other High Level Language having higher digit math accuracy and by increasing the number of loops.

V. Conclusion

It is concluded that the value of π can be calculated by simple algorithm using the properties of equilateral triangle and Pythagoras Theorem.